MHF4U1

Practice Final Exam

Section	Marks Available	Marks Earned
Section 1 – Multiple Choice	50	
Section 2 – Short Answer	56	
TOTAL	106	

SECTION 1: MULTIPLE CHOICE – 50 Marks
(Circle answers on your SCANTRON CARD)

1. Determine the (i) amplitude, (ii) period, and (iii) phase shift of the graph of $y=-2\sin 3\left(x+\frac{\pi}{4}\right)$.

(a) (i) 2 (ii) 3 (iii) $\frac{\pi}{4}$

(b) (i) -2 (ii) 2π (iii) - $\frac{\pi}{4}$

(c) (i) -2 (ii) $\frac{2\pi}{3}$ (iii) - $\frac{\pi}{4}$

(d) (i) 2 (ii) $\frac{2\pi}{3}$ (iii) - $\frac{\pi}{4}$

(e) (i) 2 (ii) $\frac{2\pi}{3}$ (iii) $\frac{\pi}{4}$

2. Determine the remainder when $2x^3+3x^2+1$ is divided by $x+1$.

(a) -2 (b) 0 (c) 2 (d) 4 (e) 6

3. Evaluate $\cos\frac{3\pi}{4}$ exactly

(a) $\frac{-1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{-1}{2}$ (d) $\frac{\sqrt{3}}{2}$ (e) $\frac{-\sqrt{3}}{2}$

4. The y-intercept of the graph of $y=k(x-1)(x-2)(x+3)$ is 24. Determine the value of k.

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

5. Write $2\log 3-\log 7-\log 11$ as a single logarithm.

(a) $\log\left(\frac{99}{7}\right)$ (b) $\log\left(\frac{63}{7}\right)$ (c) $\log\left(\frac{77}{9}\right)$ (d) $\log\left(\frac{7}{99}\right)$ (e) $\log\left(\frac{9}{77}\right)$

6. Convert 1600º to radian measure, in terms of π radians.

(a) $\frac{24\pi}{7}$ (b) $\frac{50\pi}{3}$ (c) $\frac{80\pi}{9}$ (d) $\frac{48\pi}{5}$ (e) $\frac{95\pi}{12}$

7. One factor of x^3-x^2-x+1 is $x-1$. The other factor is:

(a) x^2-x-1 (b) x^2-1 (c) x^2+1 (d) x^2+x-1 (e) x^2-x+1

8. Given that $f(x)=\frac{2x}{3}$ evaluate $f^{-1}(6)$

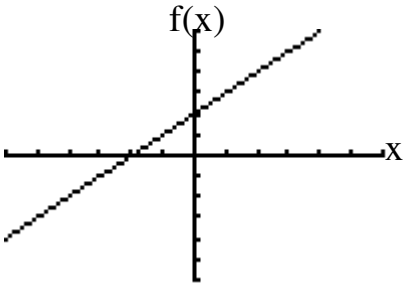
(a) 2 (b) 4 (c) 6 (d) 9 (e) 12

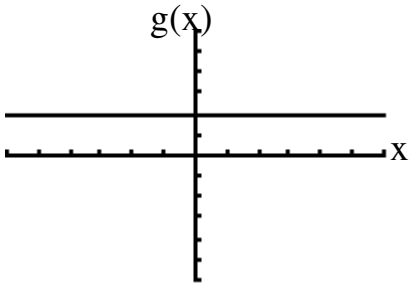
9. Which of the functions listed below do not have a vertical asymptote?

(i) $y=x^2$
(ii) $y=\log x$
(iii) $y=2^x$
(iv) $y=\tan x$
(v) $y=\frac{1}{x}$

(a) i only (b) iii only (c) i and iii only (d) i, iii and iv only (e) ii, iv and v only

10. Using the graphs shown, determine the value of $f(g(1))$.





(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

11. The interval or intervals for which $(x-1)(x-2)\rangle 0$ is/are:

(a) $1\langle x\langle 2$ (b) $x\langle 1$ or $x\rangle 2$ (c) $x\rangle 1$ (d) $x\langle 2$ (e) $x\rangle 2$

12. A certain polynomial function is represented by the following:

x	f(x)
0	0
1	2
2	10
3	30
4	68

Determine whether f is
(a) linear (b) quadratic (c) cubic (d) quartic (e) none of these

13. Which of the following is a vertical asymptote of the graph of $y=\cot x$?

(a) $x=\pi$	(b) $x=\frac{\pi}{4}$	(c) $x=\frac{\pi}{3}$	(d) $x=\frac{\pi}{2}$	(e) $x=\frac{\pi}{6}$

14. Which statement is true for the function $f(x)=2\cos x+1$?

(a) $0\le f(x)\le 3$
(b) $-1\le f(x)\le 3$
(c) $-2\le f(x)\le 1$
(d) $-3\le f(x)\le 0$
(e) $-2\le f(x)\le 2$

15. A sinusoidal function of the form $y=\sin(kx)$ has a period of 2. What is the value of ***k***?

(a) 2 (b) 2π (c) π (d) $2/\pi$ (e) $\pi/2$

16. If $\sin\theta=\cos\theta=\frac{1}{\sqrt{2}}$, determine the value of $\sin(2\theta)$.

(a) 0 (b) 1 (c) 2 (d) $\sqrt{2}$ (e) $\sqrt{2}/2$

17. Determine the vertical asymptotes of the graph of $y=\frac{1}{x^2+x-6}$.

(a) $x=1, x=-5$
(b) $x=-1, x=5$
(c) $x=3, x=-2$
(d) $x=-3, x=2$
(e) there are none

18. Solve the equation $\log_2(x+1)=3$

(a) 5 (b) 6 (c) 7 (d) 8 (e) 9

19. The equivalent exponential form of $a=b\log_c d$

(a) $b^a=c^d$ (b) $a^c=b^d$ (c) $a^c=d^b$ (d) $c^a=b^d$ (e) $c^a=d^b$

20. A Helium balloon is rising according to the function $H(t)=20\log(t+1)$ where ***H(t)*** is the height in metres after ***t*** seconds. Determine the average rate of increase of height from ***t*****=**9 seconds to ***t*****=**99 seconds.

(a) $\frac{1}{9}m/s$ (b) $\frac{2}{9}m/s$ (c) $\frac{1}{3}m/s$ (d) $\frac{4}{9}m/s$ (e) $\frac{5}{9}m/s$

21. Given that $f(x)=x^3-x^2+5x$ and $g(x)=x^3+5x^2-2$ determine $(g-f)(x)$.

(a) $4x^2+5x-2$
(b) $-6x^2+5x+2$
(c) $6x^2-5x-2$
(d) $-6x^2+5x-2$
(e) $6x^2-5x+2$

22. Given that $f(x)=3x^2$ and $g(x)=2x+1$ then $f(g(x))$ equals:

(a) $6x^3+3x^2$
(b) $6x^2+1$
(c) $4x^2+4x+1$
(d) $3x^2+2x+1$
(e) none of these

23. Using interval notation, the domain of ***f*** is [-3,5] and the domain of ***g*** is [-2,7]. On what domain is ***f+g*** defined?

(a) [-3,7] (b) [-2,5] (c) [-3,-2] (d) [5, 7] (e) [-2, 7]

24. What is the end behaviour of the graph of $y=-2^{-x}$?

(a) As $x\to\infty, y\to\infty$ and as $x\to-\infty, y\to\infty$
(b) As $x\to\infty, y\to 0$ and as $x\to-\infty, y\to-\infty$
(c) As $x\to\infty, y\to\infty$ and as $x\to-\infty, y\to 0$
(d) As $x\to\infty, y\to 0$ and as $x\to-\infty, y\to\infty$
(e) As $x\to\infty, y\to-\infty$ and as $x\to-\infty, y\to 0$

25. Solve the following equation to 3 decimal places

$$3^x=10$$

(a) 1.820 (b) 1.922 (c) 1.971 (d) 2.034 (e) 2.096

26. Evaluate $\log_2(\sec 1.2)$ to 3 decimal places.

(a) 0.392 (b) 1.465 (c) 2.033 (d) 3.909 (e) 4.118

27. The domain of $y=\log(9-x^2), x\in R,$ is

(a) $x\rangle 0$
(b) $x\rangle 3$ or $x\langle -3$
(c) $-3\le x\le 3$
(d) $x\ge 3$ or $x\le -3$
(e) $-3\langle x\langle 3$

28. One factor of $8x^3+1$ is $2x+1$. The other factor is:

(a) $2x-1$
(b) $4x^2+2x+1$
(c) $4x^2-2x+1$
(d) $4x^2-1$
(e) none of these

29. The maximum number of zeros of a quartic function is

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

30. The minimum number of zeros of a cubic function is:

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

31. The exact number of zeros of $y=\sin 2x, 0\le x\le 2\pi$ is:

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

32. Determine the quotient when x^3+2x^2-3x-1 is divided by $x+1$:

(a) x^2+x-4
(b) x^2+x+4
(c) x^2-x+4
(d) x^2-x-4
(e) none of these

33. For a car traveling at a constant speed, the distance driven, ***d*** kilometers, is represented by $d(t)=80t$, where ***t*** is the time in hours. The cost of gasoline, in dollars, for the drive is represented by $C(d)=0.09d.$ Evaluate C(d(10)).

(a) 38 (b) 44 (c) 72 (d) 80 (e) 96

34. Which of the following is true given that

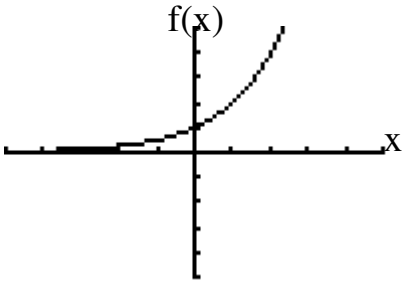
$$2(3)^a = 6\left(\frac{1}{3}\right)^b$$

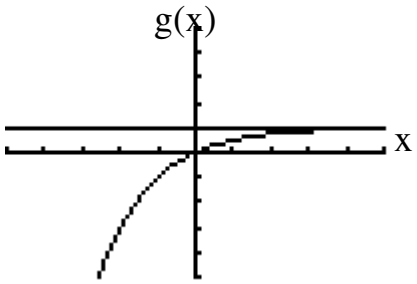
(a) $a-b=1$ (b) $a+b=1$ (c) $b-a=1$ (d) $ab=1$ (e) $a/b=1$

35. An expression identical to $\sin\left(x+\frac{\pi}{2}\right)$ is

(a) – cos x
(b) sin x + cos x
(c) sin x
(d) cos x
(e) – sin x

36. Use the graphs below to determine the relationship between the two functions:





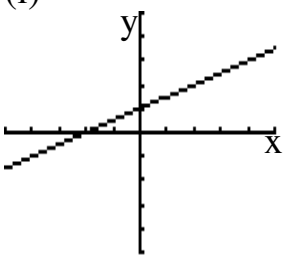
(a) $g(x)=\log_2(x+1)$
(b) $g(x)=-f(x)$
(c) $g(x)=f(-x)+1$
(d) $g(x)=-f(-x)-1$
(e) $g(x)=-f(-x)+1$

37. Which of the following is closest in value to 3 radians?

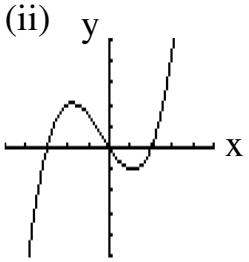
(a) 150º (b) 160º (c) 170º (d) 180º (e) 190º

38. Which of the following functions have inverses that are also functions?

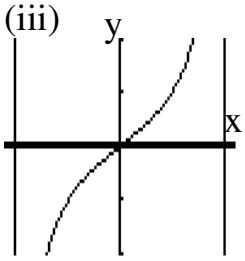
(i)



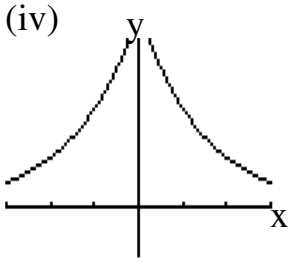
(ii)



(iii)



(iv)

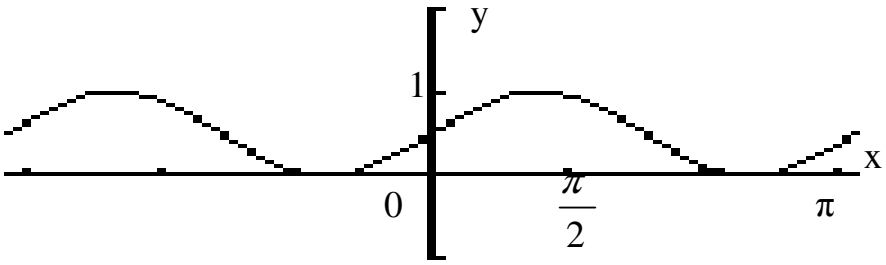


(a) (i) , (ii), and (iii) only
(b) (i) and (iii) only
(c) (ii) and (iv) only
(d) (iii) only
(e) All of them

39. Solve $3\cos x - 1 = 0, 0 \le x \le \frac{\pi}{2}$ to three decimal places

(a) 1.231 (b) 1.383 (c) 1.402 (d) 1.413 (e) none of these

40. Which equation models the following graph for $-\pi \le x \le \pi$?



(a) $y = 0.5\sin(0.5x) + 0.5$
(b) $y = 0.5\sin x + 0.5$
(c) $y = 0.5\sin 2x + 0.5$
(d) $y = 0.5\cos 2x + 0.5$
(e) $y = -0.5\sin 2x + 0.5$

41. The amount of money in dollars, A(t) after ***t*** years is given by the function $A(t)=1000(1.045)^t$. What is the amount after 10 years?

(a) 1142.93 (b) 1198.04 (c) 1351.32 (d) 1552.97 (e) 1572.63

42. The value of $\log_a(\log_a a)$ is:

(a) 0 (b) 1 (c) a (d) a^2 (e) none of these

43. Solve $2\sin x+\sqrt{3}=0, 0\le x\le 2\pi$.

(a) $\left\{\frac{\pi}{4},\frac{3\pi}{4}\right\}$

(b) $\left\{\frac{\pi}{3},\frac{2\pi}{3}\right\}$

(c) $\left\{\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

(d) $\left\{\frac{4\pi}{3},\frac{5\pi}{3}\right\}$

(e) none of these

44. The position of a particle on a spinning wheel is given by the function $f(t)=5\cos 200t$, where ***t*** is measured in seconds. How many seconds will it take the wheel to spin 50 revolutions?

(a) 2π (b) π (c) $\frac{\pi}{2}$ (d) $\frac{1}{\pi}$ (e) $\frac{2}{\pi}$

45. Determine the instantaneous rate of change of $y=\tan x$ at $x=1$ to 3 decimal places:

(a) 1.317 (b) 2.509 (c) 3.426 (d) 3.652 (e) 4.029

46. A bacteria culture is growing according to the function $N(t)=100(2)^t$, where ***N(t)*** is the population after ***t*** hours. How long will it take the population to grow to 51200?

(a) 8 h (b) 9 h (c) 10 h (d) 11 h (e) 12 h

47. Given that $f(x)=x-2$ and $g(x)=x^2+3x-10$ determine $\left(\frac{f}{g}\right)(x)$.

(a) $x+5$ (b) $x-5$ (c) $\frac{1}{x+5}$ (d) $\frac{1}{x-5}$ (e) none of these

48. Simplify the function $y=4(2)^x$

(a) $y=2^{3x}$ (b) $y=2^{x+4}$ (c) $y=2^{2x}$ (d) $y=2^{x^2}$ (e) $y=2^{x+2}$

49. The graph shown below is the reciprocal of what function?

(a) $y=\sin x$ (b) $y=\cos x$ (c) $y=\tan x$ (d) $y=\sec x$ (e) $y=\csc x$

50. The horizontal asymptote of the graph of $y=\frac{2x^2-1}{x^2+3}$ is

(a) $y=0$ (b) $y=1$ (c) $y=2$ (d) $y=-1/3$ (e) $y=-3$

SECTION 2: SHORT ANSWER – 56 Marks
Place final answer in the box. Use the space to show work.

51. Simplify the rational expression $\frac{x^4+3x^3-2x^2-6x}{x^3-2x}$ stating all restrictions on the variable.

[3]

52. Determine the zeros of the function $f(x)=3(x^2-25)(4x^2+4x+1)$.

[2]

53. The remainder when x^4+Cx^2 is divided by $x+2$ is 28. Determine the value of ***C***.

[2]

54. Divide x^3+x+1 by $x+1$ and write the answer in the form $Q(x)+\frac{r}{x+1}$.

[3]

55. If $\sin x=-\frac{2}{3}$ where $\frac{3\pi}{2}\langle x\langle 2\pi$, determine the exact value of $\cos 2x$.

[2]

56. The height, in metres, of a passenger on a Ferris wheel is given by the function $h(t)=11-10\cos(0.1\pi t)$, where ***t*** is the time in seconds. How long will it take a passenger to reach a height of 5 metres the first time? Express your answer to 3 decimal places.

[3]

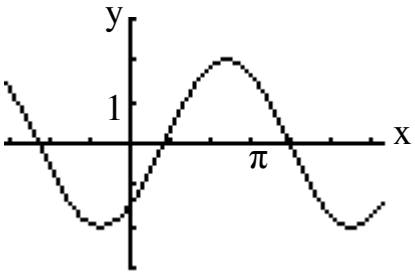
57. Determine the inverse of $y = 2(3)^x + 1$

[3]

58. Determine the exact value of $3^{-1} - 2\left(\frac{1}{8}\right)^{-\frac{2}{3}}$.

[2]

59. Write an equation of a periodic function that models the graph shown:



[2]

60. Referring to the graph in #59, state:

(a) an interval over which the function increases

(b) an overall minimum value

(c) an interval over which the second finite differences would be negative.

[3]

61. Two functions, $y = f(x)$ and $y = g(x)$ are defined by the tables given. Determine:

(a) $f(g(2))$

(b) $\dfrac{f^{-1}(4) - f^{-1}(0)}{4}$

(c) State what the answer to (b) represents

[3]

x	f(x)
-2	-4
-1	-1
0	0
1	1
2	4

x	g(x)
1	-3
2	-2
3	-1
4	0
5	1

62. Evaluate $\cos\frac{7\pi}{12}$ exactly.

[2]

63. Solve $1-\tan^2 x=0,\ 0\le x\le 2\pi$

[2]

64. Simplify the expression $\frac{\sin^4 x-\cos^4 x}{\cos 2x}$

[3]

65. Determine the vertical asymptotes of the graph of $y = \frac{\sec x}{\log x}$ for $x \leq 2\pi$.

[2]

66. For the function $f(x) = \frac{6 - 5(3)^{x+1}}{3}$,

(a) simplify

(b) state the domain

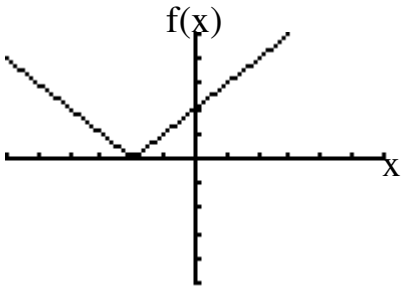
(c) state the range

[3]

67. Given the graph of a function $y = f(x)$ shown below,

(a) sketch the graph of the inverse

(b) state the restrictions on the domain of $\boldsymbol{f}$ that must be imposed in order that $\boldsymbol{f^{-1}}$ is a function.



[2]

68. Given that $f(x) = \dfrac{x^3 - 8}{x - 2}$, determine $f(a+1)$ in simplest form.

[3]

69. Solve $\log_{12}(x-2) + \log_{12}(x+2) = 1$.

[3]

70. (a) Express $y=-2(x+3)(2x-1)(x-4)$ in standard form

(b) Determine its end behaviour

[3]

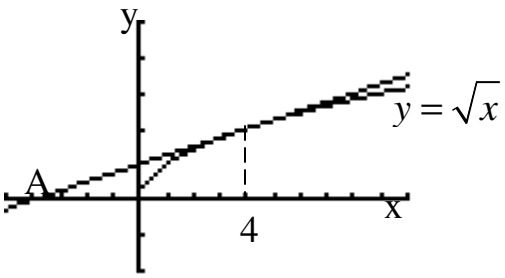
71. Simplify $\dfrac{\left(\frac{1}{2}a\right)^3}{\sqrt[3]{a}}$ and express your answer in radical form.

[3]

72. Given that $f(x)=g\left(\frac{x}{3}\right)$, describe how the graphs of ***f*** and ***g*** are related?

[2]

73. The tangent line to the curve $y=\sqrt{x}$ at $x=4$ has a slope of $\frac{1}{4}$. Determine the coordinates of ***A*** shown.



[3]

ANSWERS:

1. D	11. B	21. C	31. E	41. D
2. C	12. C	22. E	32. A	42. A
3. A	13. A	23. B	33. C	43. D
4. D	14. B	24. B	34. B	44. C
5. E	15. C	25. E	35. D	45. C
6. C	16. B	26. B	36. E	46. B
7. B	17. D	27. E	37. C	47. C
8. D	18. C	28. C	38. B	48. E
9. C	19. E	29. E	39. A	49. A
10. E	20. B	30. B	40. C	50. C

SHORT ANSWERS:

51. $= \dfrac{x(x^3+3x^2-2x-6)}{x(x^2-2)}$

$= \dfrac{x^2(x+3)-2(x+3)}{x^2-2}, x \neq 0$

$= x+3, x \neq 0, x \neq \pm\sqrt{2}$

52. $f(x) = 3(x+5)(x-5)(2x+1)^2$

$x = -5, -\dfrac{1}{2} or 5$

53. By theorem ,

$f(-2) = 28$

$\therefore (-2)^4 + c(-2) = 28$

$c = 3$

54. $x+1 \overline{)\, x^3+0x^2+x+1}$ $\quad x^2-x+2$

x^3+x^2

$-x^2+x$

$-x^2-x$

$2x+1$

$2x+2$

-1

$\therefore x^2-x+2-\dfrac{1}{x+1}$

55.

$x^2 + 4 = 9$

$x = \sqrt{5}$

$\cos x = \frac{\sqrt{5}}{3}$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = \frac{1}{9}$

OR

$\cos 2x = 1 - \sin^2 x$

$\cos 2x = 1 - \frac{8}{9}$

$\cos 2x = \frac{1}{9}$

56. $11 - 10\cos(0.1\pi t) = 5$

$\cos(0.1\pi t) = 0.6$

$t = \frac{\cos^{-1}(0.6)}{0.1\pi}$

$t = 2.952 \sec.$

57. The inverse is $x = 2(3)^y + 1$

$\frac{x-1}{2} = 3^y$

$y = \log_3\left(\frac{x-1}{2}\right)$

58. $= \frac{1}{3} - 2(8)^{\frac{2}{3}}$

$= \frac{1}{3} - 8$

$= -\frac{23}{3}$

59. Answers may vary. $y = 2\sin\left(x - \pi/4\right)$ or $y = 2\cos\left(x - 3\pi/4\right)$

60. Answers may vary,

a) $-\frac{\pi}{4} \langle x \langle \frac{3\pi}{4}$

b) -2

c) $\frac{\pi}{4} \langle x \langle \frac{5\pi}{4}$

61. a) -4

b) ½

c) the average rate of change of $y = f^{-1}(x)$ over $0 \le x \le 4$

62. $\cos\left(\frac{3\pi}{12}+\frac{4\pi}{12}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{3}-\sin\frac{\pi}{4}\sin\frac{\pi}{3}=\frac{1}{\sqrt{2}}\bullet\frac{1}{2}-\frac{1}{\sqrt{2}}\bullet\frac{\sqrt{3}}{2}=\frac{1-\sqrt{3}}{2\sqrt{2}}$

63. $\tan x=\pm 1.\quad x=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

64. $\frac{(\sin^2 x-\cos^2 x)(\sin^2 x+\cos^2 x)}{\cos^2 x-\sin^2 x}=-1$

65. $y=\frac{1}{\cos x\log x}$

$\cos x\neq 0,\log x\neq 0$

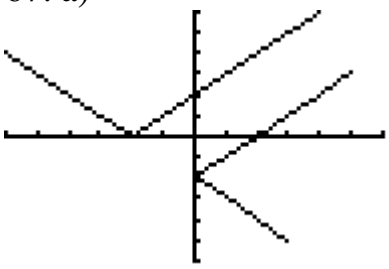
$x=0,\frac{\pi}{2},\frac{3\pi}{2}$

66. a) $2-5(3)^x$

b) $\{x \mid x\in R\}$

c) $\{y \mid y\leq 2, y\in R\}$

67. a)



b) $x\rangle -2$ or $x\langle -2$

68. $f(x)=\frac{(x-2)(x^2+2x+4)}{(x-2)}=x^2+2x+4$

$f(a+1)=(a+1)^2+2(a+1)+4=a^2+4a+7$

69. $\log_{12}(x^2-4)=1$

$x^2=16$

$x=\pm 4$

$x=4$

70. $y = -2(2x^2 + 5x - 3)(x - 4)$

$y = -2(2x^3 - 3x^2 - 23x + 12)$

a) $y = -4x^3 + 6x^2 + 46x - 24$

b) As $x \to -\infty, y \to \infty$. As $x \to \infty, y \to -\infty$

71. $\dfrac{\left(\frac{1}{2}a\right)^3}{\sqrt[3]{a}} = \dfrac{\frac{1}{8}a^3}{a^{1/3}} = \dfrac{1}{8}a^{8/3} = \dfrac{\sqrt[3]{a^8}}{8}$

72. The graph of ***f*** is the graph of ***g*** stretched horizontally by a factor of 3.

73. $y = 2$. Using $y = mx + b \ldots$

$2 = \dfrac{1}{4}(4) + b$

$1 = b$

Therefore, the tangent line is $y = \dfrac{1}{4}x + 1$ when y=0, x=-4