AChor/MHF4U **Name:** _____
Date: _____

Worksheet 1-5: Slopes of Secants and Average Rate of Change

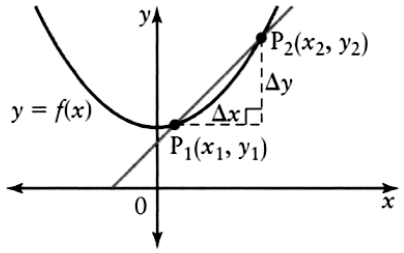
A rate of change is a measure of the change in one quantity (the dependent variable) with respect to a change in another quantity (the independent variable).

There are two types of rates of change, **average** (this lesson) and **instantaneous** (next lesson).

An average rate of change is a change that takes place over an interval, while an instantaneous rate of change is a change that takes place in an instant.

Key Concepts: Slopes of Secants and Average Rate of Change

- A rate of change is a measure of how quickly one quantity (the dependent variable) changes with respect to another quantity (the independent variable).
- Average rates of change
  - represent the rate of change over a specified interval
  - correspond to the slope of a secant between two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on a curve



$$\text{Average rate of change} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

- An average rate of change can be determined by calculating the slope between two points given in a table of values or by using an equation.

Describe a situation for which the average rate of change is

a) constant and positive **b)** constant and negative **c)** zero

State the average rate of change for this situation. When the change in the independent variable is -3, the change in the dependent variable is 12.

a) What information is provided by the sign (positive or negative) of an average rate of change?

b) How can you tell from a graph if the average rate of change over an interval is positive or negative? Is it always possible to do so? Explain.

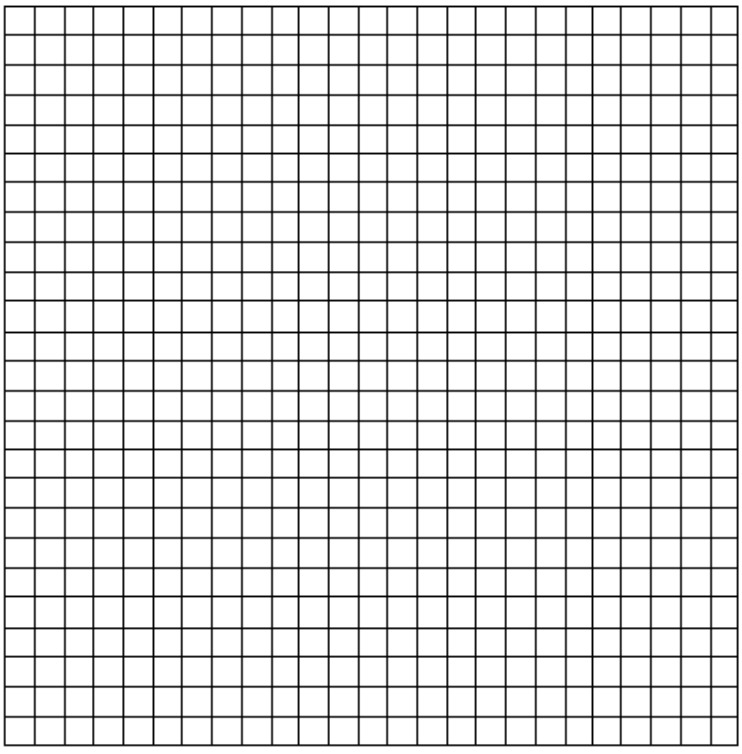
1. Seismic activity at a certain point on the ocean floor creates a wave that spreads in a circular pattern over the calm surface of the ocean. The table shows the radius of the circular pattern during the first 10 s as the wave moves outward.

Time, t (s)	Radius, r (m)
0	0
1	2
2	4
3	6
4	8
5	10
6	12
7	14
8	16
9	18
10	20

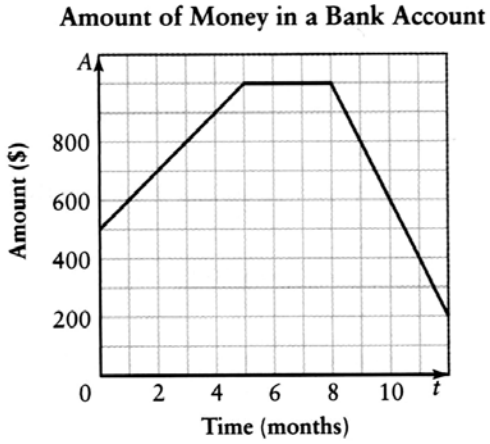
(a) Identify the independent variable and the dependent variable. Justify your choice.

(b) Determine $\frac{\Delta r}{\Delta t} = \frac{\text{Change in radius}}{\text{Change in time}}$ for each interval: (i) [0, 10] (ii) [0, 1] (iii) [9, 10].

(c) Graph the data. What type of polynomial function does the graph represent? Explain.



2. The graph represents the amount of money in a bank account over a 1-year period.



(a) How much money was in the account

(i) at the start of the year

(ii) at the end of the year

(b) What does the graph tell you about the average rate of change of the amount of money in the account in the following intervals:

(i) month 0 to month 5

(ii) month 5 to month 8

(iii) month 8 to month 12

(c) Determine the average rate of change for the time periods in part (b). Interpret these values in this situation.

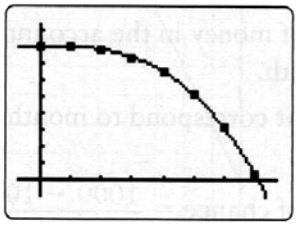
(i) month 0 to month 5

(ii) month 5 to month 8

(iii) month 8 to month 12

3. A new antibacterial spray is tested on a bacterial culture. The table shows the population, P, of the bacterial culture t minutes after the spray is applied.

t (min)	P
0	800
1	799
2	782
3	737
4	652
5	515
6	314
7	37



(a) How can you tell that the average rate of change is negative by examining

(i) the table of value

(ii) the graph

(b) Determine the average rate of change of the number of bacteria over the entire time period shown in the table. Interpret this value for this situation.

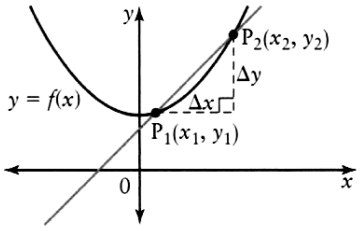
(c) How can you tell that this situation involves a non-constant rate of change by examining

(i) the table of value

(ii) the graph

(iii) the average rate of change

The average rate of change between two points corresponds to the slope of the secant between the points. For example, the average rate of change of y with respect to x between the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ is determined as follows:



$$\text{Average rate of change} = \frac{\Delta y}{\Delta x}$$
$$= \frac{\text{change in } y}{\text{change in } x}$$
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

4. A football is kicked into the air such that its height, *h*, in metres, after *t* seconds can be modelled by the function $h(t) = -4.9t^2 + 14t + 1$.

(a) Determine the average rate of change of the height of the ball for each time interval.

(i) [0, 0.5]

(ii) [2. 2.5]

(b) Consider the graph of $h(t) = -4.9t^2 + 14t + 1$ with secant lines AB and CD. Describe the relationship between the values in part (a), the secant lines, and the graph.