AChor/MHF4U

Name: ______________________
Date: ______________________

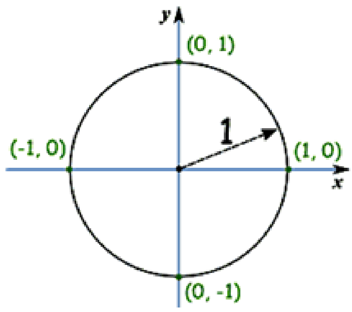
Worksheet 4-2: Trigonometric Ratios and Special Angles in Radians

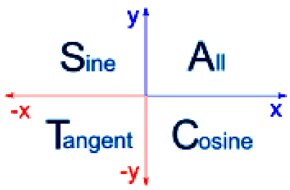
Key Concepts of Calculating Trigonometric Ratios and Special Angles in Radians:

- You can use a calculator to calculate trigonometric ratios for an angle expressed in radian measure by setting the angle mode to radians.
- You can determine the reciprocal trigonometric ratios for an angle expressed in radian measure by first calculating the primary trigonometric ratios and then using the reciprocal key on a calculator.
- You can use the unit circle and special triangles to determine exact values for the trigonometric ratios of the special angles 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$.
- You can use the unit circle along with the CAST rule to determine exact values for the trigonometric ratios of multiples of the special angles.

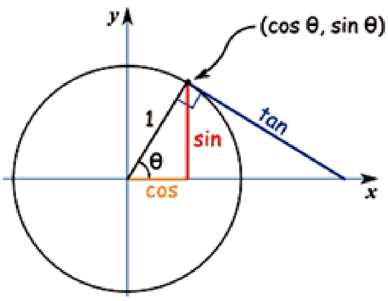
The Unit Circle

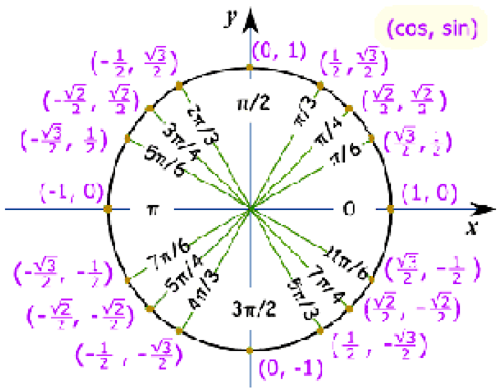
The "Unit Circle" is just a circle with a radius of 1.





Because the radius is 1, you can directly measure sine, cosine and tangent.





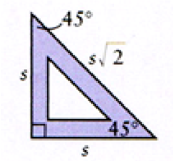
AChor/MHF4U

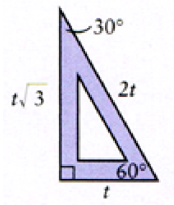
Name: ____________________ WS 4-2

Date: ____________________

Special Angles: 45°–45°–90° Triangle and 30°–60°–90° Triangle

The triangles found in geometry set are a 45°–45°–90° triangle and a 30°–60°–90° triangle. These triangles can be used to construct similar triangles with the same special relationships among the sides.





Practice 1: Apply Trigonometric Ratios for Special Angles

Ravinder is flying his kite at the end of a 50-m string. The sun is directly overhead, and the string makes angle of $\frac{\pi}{6}$ with the ground. The wind speed increases, and the kite flies higher until the string makes an angle of $\frac{\pi}{3}$ with the ground.

(a) Determine an exact expression for the horizontal distance that the shadow of the kite moves between the two positions of the kite. (Hint: Draw a diagram to represent the situation.)

$$\frac{\pi}{6} \times \frac{180°}{\pi} = \frac{180°\pi}{6\pi} = 30°$$

$$\frac{\pi}{3} \times \frac{180°}{\pi} = \frac{180°\pi}{3\pi} = 60°$$

P_1

cos of P_1 (adj)

$\cos\frac{\pi}{6} = \frac{x_1}{50}$ ← hyp

$\frac{\sqrt{3}}{2} = \frac{x_1}{50}$

$\frac{50\sqrt{3}}{2} = x_1$

$25\sqrt{3} = x_1$

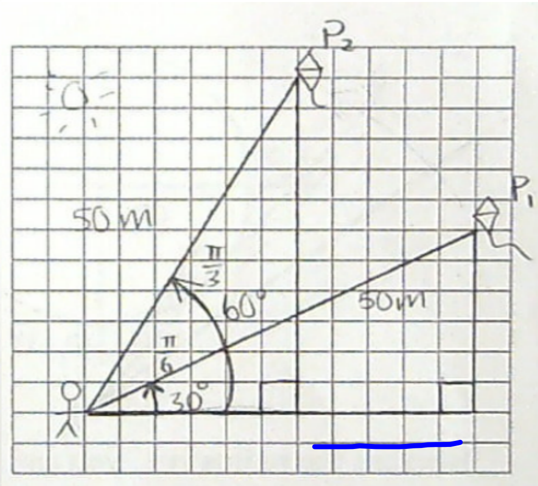
P_2

$\cos\frac{\pi}{3} = \frac{x_2}{50}$

$\frac{1}{2} = \frac{x_2}{50}$

$50\left(\frac{1}{2}\right) = x_2$

$25 = x_2$



Difference $= 25\sqrt{3} - 25 = 25(\sqrt{3}-1)$

(b) Determine the distance in part (a), to the nearest tenth of a metre. Answer statement

$25(\sqrt{3}-1) \doteq 18.3$ m

The horizontal distance between the two positions is 18.3m.

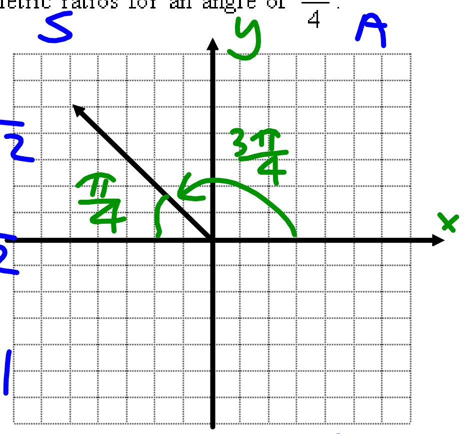
AChor/MHF4U

Name: ____________________ WS 4-2
Date: ____________________

Practice 2: Trigonometric Ratios for a Multiple of a Special Angle

Use the unit circle to determine exact values of the six trigonometric ratios for an angle of $\frac{3\pi}{4}$.

(Hint: Sketch the angle in standard position.)



$\cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$

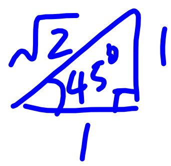
$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

$\tan\frac{3\pi}{4} = -1$

$\sec\frac{3\pi}{4} = -\sqrt{2}$

$\csc\frac{3\pi}{4} = \sqrt{2}$

$\cot\frac{3\pi}{4} = -1$



* Rationalize our answers !

Practice 3: Determine Exact or Approximate Values of Trigonometric Expressions

Determine an exact value for each expression, and use a calculator to check your answers.

(a) $\dfrac{\cos\frac{4\pi}{3}\tan\frac{5\pi}{6}}{\sin\frac{3\pi}{4}}$

$= \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)}{\left(\frac{1}{\sqrt{2}}\right)}$

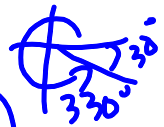
$= \left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)\left(\sqrt{2}\right)$

$= \frac{\sqrt{2}}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{6}}{6}$

(b) $\cot\frac{5\pi}{4} + \tan\frac{11\pi}{6}\tan\frac{5\pi}{3}$

$\frac{\pi}{3}$



$= 1 + \left(-\frac{1}{\sqrt{3}}\right)\left(-\sqrt{3}\right)$

$= 1 + 1$

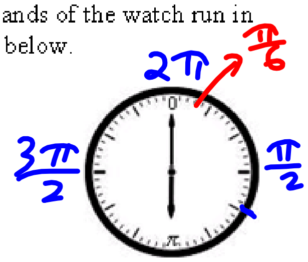
$= 2$

ACharMHF4U

Name: ______________ WS 4-2
Date: ______________

Practice 4: Problem Solving with Radian Measure

When Sunita received her bachelor's degree in mathematics, her friends presented her with a "radian watch". They repainted the face of the watch. Instead of the usual numbers around the face, they replaced the 12 with 0 and the 6 with π. The hour and minute hands of the watch run in the usual clockwise direction. A radian time of π is shown in the diagram below.



(a) What radian time corresponds to 3:00?

$\frac{\pi}{2}$

(b) What radian time corresponds to 4:00?

$\frac{\pi}{2} + \frac{\pi}{6} = \frac{4\pi}{6} = \frac{2\pi}{3}$

(c) What normal time corresponds to a radian time of $\frac{3\pi}{2}$?

9:00

(d) What normal time corresponds to a radian of $\frac{11\pi}{6}$?

11:00

(e) What radian time corresponds to 7:30? 30 min $= \frac{\pi}{12}$ 1 hr $= \frac{\pi}{6}$

$7\left(\frac{\pi}{6}\right) + \frac{\pi}{12} = \frac{14\pi + \pi}{12} = \frac{15\pi}{12} = \frac{5\pi}{4}$

5. Does $\sin\frac{\pi}{2} = \frac{1}{2}\sin\pi$? Explain why or why not?

$\sin\frac{\pi}{2} = 1$ | $\frac{1}{2}\sin\pi = \frac{1}{2}(0) = 0$

$\therefore \sin\frac{\pi}{2} \neq \frac{1}{2}\sin\pi$

Think of $\sin\frac{1}{2}x$ (horizontal expansion) vs. $\frac{1}{2}\sin x$ (vertical compression)

Assigned work: P.216-18 #3-6, 8-11, 13, 20

AChor/MHF4U

Name: ______________________
Date: ______________________